Decentralized Distributed Optimization for Sum of Convex Functions

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Problem Definition

Solve the optimization problem

$$\min_{x \in \mathrm{R}^n} \sum_{i=1}^{m} f_i(x)$$

where $\{f_i \ : \ \mathrm{R}^n \mapsto \mathrm{R}\}$ are convex functions,

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$$\min_x \|Ax - b\|_2^2 + \|x\|_2^2,$$

and support vector machine,

$$\min_x \sum_{i=1}^{m} \max\left(0, 1 - y_i \langle \theta_i, x \rangle\right) + \|x\|_1.$$

Make problem more independent

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Alternatively,

$$\min_{x_1, x_2, \ldots, x_m \in \mathrm{R}^n} \sum_{i=1}^{m} f_i(x_i)$$
$$\text{s.t.} \quad x_1 = x_2 = \ldots = x_m.$$

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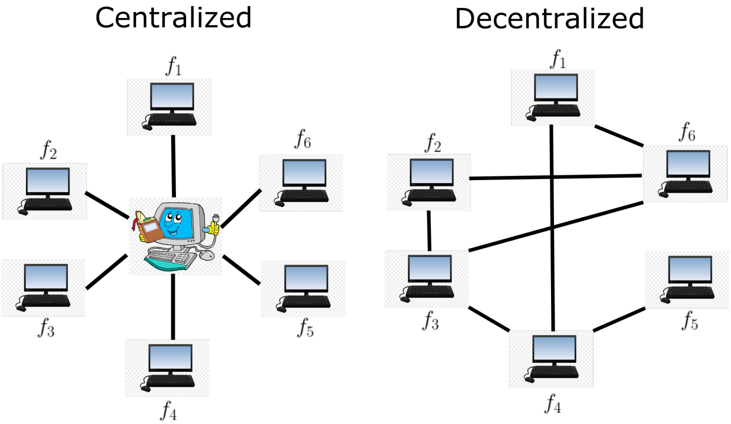
Write as unconstrainted optimization

$$\min_{[x_1, \ldots, x_m] = \boldsymbol{x} \in \mathrm{R}^{m \cdot n}} \sum_{i=1}^{m} f_i(x_i) + g(\boldsymbol{x}),$$

where g is smooth function enforcing consensus, $x_1 = x_2 = \ldots = x_m$.

Centralized vs. Decentralized

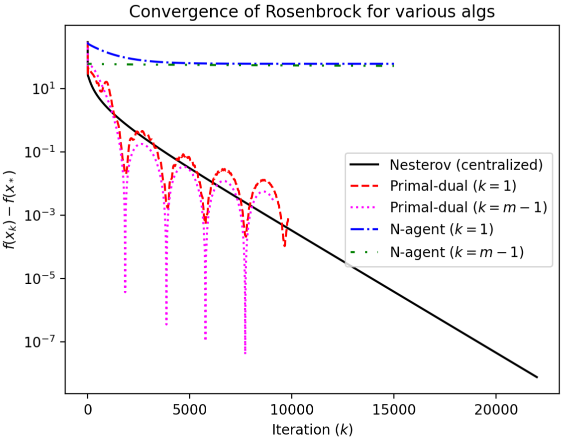
$$\min_x f_1(x) + f_2(x) + f_3(x) + f_4(x) + f_5(x) + f_6(x)$$



Numerical Results

Solve Rosenbrock function

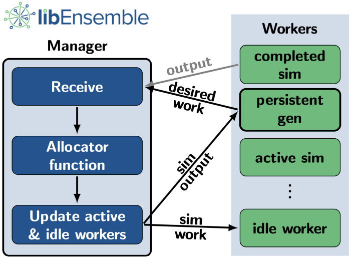
$$\min_{x \in \mathbb{R}^n} f(x) = \sum_{i=1}^{n/2} \underbrace{\left(100\left(x_{2i-1}^2 - x_{2i}\right)^2 + (x_{2i-1} - 1)^2\right)}_{f_i(x)},$$



Conclusion

Last steps

1. Test more algorithms and problems
2. Finish implementation in libEnsemble



The End

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